Black Holes and Quantum Gravity at the LHC

LR with Patrick Meade
One of the most anticipated signals at the LHC has been small higher-dimensional black holes. If higher-dimensional gravity/Planck scale is ~TeV, center of mass collisions at LHC energies will often exceed TeV. Superficially, conditions will be met for small higher-dimensional black hole formation.
Black hole formation

- Two particles collide
- When energy is big enough, it gets trapped inside Schwarzschild radius
- Black hole is formed!
Appears Very Promising

- Estimate black hole production cross section

\[ \sigma(E) \sim \pi R_S(E)^2 \]

\[ \sigma(E) \sim \frac{1}{M^2} \left( \frac{E}{M} \right)^\alpha \]

M~TeV=>~100 pb cross section

Not suppressed by gauge couplings or phase space factors

Original claim:

Prolific Production!
Signature

- Claim was multiparticle final state
- Spherically distributed: particles in all directions
- Characteristic of Hawking radiation

Spectacular fireball final states!
This Talk

- How much of this is true?
  - Do we even produce black holes?
- We will see:
  - LHC unlikely to make classical black holes
  - Multiparticle final states that decay with high multiplicity via Hawking radiation even more unlikely
However...all is not lost

- Potentially much more prolifically produced 2 body final states
- Uncalculable, but we will see distinctive experimental signatures that will distinguish among models
- Might teach us about quantum gravity
Remember...

- Challenges as LHC approaches
  - Are we optimizing existing searches?
  - Are we doing all the searches possible?
- Focus on extra dimensions and low scale gravity shows new searches important for strongly interacting physics
Preliminaries

- We’ll consider ADD and RS type black holes
- **ADD** (Arkani-Hamed, Dimopoulos, Dvali)
  - Large extra dimensions
  - Low quantum gravity scale
  - $M_{pl}^2 = M^2 + n V$
    - Experimental bound strong for low $n$
    - Comes from KK mode production
    - In spirit of generosity, we’ll calculate for least constrained case of $n=6$
    - $M_D > 900$ GeV
RS (Randall Sundrum)
- Warped extra dimension
- Low quantum gravity scale for five-dimensional theory near Weakbrane
- $M \sim e^{-kr} M_{Pl}$

Not usually considered for phenomenological black holes
- People consider either
  - Pancake (UV)
  - Strong bound states characteristic of AdS
- However, for $k < M$, regime $M < M_{BH} < (M/k)^2 k$
- Traditional 5d (almost) flat space black holes
- $M > 500$ GeV
RS Flat Space Black Holes

How to see this?

- Just use metric with $\tilde{M}$ as fundamental variable
- We are just interested in TeV physics here...
- Use conformal coordinates where flat space metric is manifest

$$ds^2 = \frac{1}{(kz)^2} (dz^2 + dx^2)$$

And find gravitational action

$$M^3 \left(\frac{1}{kz_0}\right)^3 \int d^4xdz \sqrt{g}R = \tilde{M}^3 \int d^4xdz \sqrt{g}R$$

- Use original coordinates and match to classical potential

$$V(r) \sim \frac{1}{M_{Pl}^2} \frac{m_am_b}{r} + \frac{1}{M_{Pl}^2k} \frac{m_am_be^{-m_1r}}{r^2}$$

$$r_s = \left(\frac{M_{BH}}{3\pi^2\tilde{M}^3}\right)^{1/2}$$
Why so different?
Estimate was always optimistic!

- At what energy is black hole created?
- Clearly $E > M$, but vague and insufficient
  - PDFs drop rapidly
  - Quarks, gluons carry only a fraction of the energy of protons
  - Rate depends dramatically on threshold
  - Important since we are necessarily near black hole production border

- Terms in original estimate must be considered carefully
  - $M$: quantum gravity scale
  - $M_{BH}$: black hole mass relative to center of mass energy
I: “M (gravity scale)” -- Convention Dependent

**Myers-Perry Convention:**

\[
\frac{1}{16\pi G_D} \int d^{D+1} x \sqrt{g} R
\]

\[
r_s = \left( \frac{M_{BH}}{\mathcal{L}_N 6\pi^2} \right)^{1/2}
\]

**Different Normalizations for G_D:** eg for 5d:

\[
M_P^3/16\pi \text{ with the convention used in [1], } M^3/2 \text{ with the RS convention, and } M_D^3/4\pi
\]

**Convention-dependent Schwarschild Radius:**

\[
r_s = \left( \frac{M_{BH}}{M^3 3\pi^2} \right)^{1/2}
\]

\[
r_{s, \text{dimopoulous}} = \left( \frac{8 M_{BH}}{M_P^3 3\pi} \right)^{1/2}
\]

\[
r_{s, \text{feng}} = \left( \frac{2 M_{BH}}{M_D^3 3\pi} \right)^{1/2}
\]

\[
M_P, 1.6 \ M_D, 2.9 \ M
\]
II: “$M_{BH}$” Thermal Black Hole Threshold?

- Quantum gravity scale—$M$—convention-dependent
  - (Physical question is black hole threshold relative to experimental bound which is convention-independent)

- Begs question: at what scale above $M$ can we safely say we are making black holes?
- How much entropy do we need?
- What is $M_{BH}^{\text{threshold}}$?
Criteria for a Black Hole?

- $M_{BH} > M$
  - As advertised, not even conventional independent
- $\frac{2\pi}{(M/2)} < R_S$
  - Compton radius inside Schwarzschild radius
  - More stringent version of above
  - ADD (n=6) $M_{BH} > 4M$—almost at experimental limit
  - RS $M_{BH} > 16M$—if taken seriously, bhs already out of reach!
Additional Criteria

- Model-dependent constraint on black hole mass vs. brane tension
- Black holes should be thermal
  - Each degree of freedom should carry small fraction of energy
    - Small change in temperature when single particle emitted
- Black hole lifetime greater than $1/M$, $R_S$
- Not sharp criteria but we can have minimal essential requirements!
Express in terms of threshold parameter

$M_{\text{threshold}} = x_{\text{min}} M$

Useful formulae:

$$r = \frac{1 + n}{4\pi T} = \frac{k(n)}{M_D} \left( \frac{M}{M_D} \right)^{\frac{1}{1+n}},$$

$$k(n) = \left(2^n \pi^{\frac{n-3}{2}} \frac{\Gamma \left( \frac{n+3}{2} \right)}{2 + n} \right)^{\frac{1}{1+n}}$$

$$S = \frac{1 + n M_{BH}}{2 + n T_{BH}}$$
Constraints on $x_{\text{min}}$

- Small back reaction on temperature. A weak constraint that is readily satisfied:
  \[ \frac{\partial T}{\partial M} \sim \frac{1}{(n+2)S} \]

- Individual degree of freedom should carry small fraction of mass: $(n+3) \frac{T}{M}$

- Black hole lifetime bigger than $\frac{1}{M}$

  **ADD:**
  \[ \tau = 0.38 \frac{x_{\text{min}}^2}{M} \]

  **RS:**
  \[ \tau = 0.7 \frac{x_{\text{min}}^2}{M_D} \]

- Really black hole lifetime greater than $R_s$

  **ADD:** $x_{\text{min}} > 3$
Number of Particles, \( N \)

- Use ratio of energy loss to particle loss
- Compute time-dependent decay including grey-body factors

\[
dE/dt \sim f(E/T) \ E \ d^4k \sim \Gamma(4)
\]
\[
dN/dt \sim f(E/T) \ d^4k \sim \Gamma(3)
\]

- Critical to computing particle number is assumption of decays on the brane
  - General \( n \) would have given \( \Gamma(n), \Gamma(n-1) \)

\[
\langle N \rangle \sim \frac{4\pi \rho k(6)}{8} \left( \frac{M_{BH}}{M_D} \right)^{3/4}
\]
\[
\rho = \frac{\sum c_i g_i \Gamma_i \zeta(3) \Gamma(4)}{\sum c_i f_i \Phi_i \zeta(4) \Gamma(4)}
\]
\[
\langle N \rangle \sim \frac{4\rho}{3\sqrt{3}} \left( \frac{M_{BH}}{M} \right)^{3/2}
\]
Not many Degrees of Freedom carrying Black Hole Energy...

- Decay dof carry $3/2$ T
- But bulk dof carry energy $(n+3)/2$ T
- Require $3/(n+3) < N >>> 1$
- $x_{\text{min}} \sim 3$ for RS, $x_{\text{min}} \sim 2$ for ADD ($n=6$);
  - But this is one dof carrying energy of bh...
- Max $x_{\text{min}}$ within reach for ADD: $x_{\text{min}} \sim 6$ yielding 3 bulk particles!
- Max $x_{\text{min}}$ for RS: $x_{\text{min}} \sim 10$ yielding about 6 particles...
Constraints: Min value of $M_{\text{BH}}$
Conclude From This

- $x_{\text{min}}$ should be reasonably high

- Furthermore, even if a black hole produced nontrivial $x_{\text{min}}$, obscures ability to extract $M$ from total cross section

- In principle, energy dependence gives number of dimensions—but tough

- Differential cross section (threshold behavior) could be used in principle to extract $M$

- But confused by inelasticity, we now discuss
What is true threshold energy? Role of Inelasticity

Black hole: energy at least $x_{\text{min}} M$

Fraction of com energy goes in bh?

- Important since PDFs fall rapidly—effectively increases threshold
- Penrose, D’eath and Payne, Eardley and Giddings, Yoshino and Rychkov
- Parameterize two Aichelberg-Sexl shock waves (two highly boosted particles) intersecting
- What fraction of energy gets trapped behind horizon?
- Classical regime...
Inelasticity Reduction

- **Without inelasticity**

\[ \sigma(pp \rightarrow X) = \sum_{i,j} \int dx_1 dx_2 f_i(x_1) f_j(x_2) \sigma(ij \rightarrow X) \]

- **With inelasticity**

- **Define** \( y = \frac{M_{BH}}{\sqrt{s}} \)

\[ \sigma(pp \rightarrow BH) \equiv \sum_{i,j} \int_0^1 2z dz \int_0^1 \frac{M_{DP}}{y(z)^2 s} du \int_u^1 dv f_i(v, Q) f_j(u/v, Q) \sigma_{ij \rightarrow BH}(M_{BH} = us) \]
Inelasticity (as function of impact parameter) is significant

From Yoshino and Rychkov
\[ \sigma(pp \rightarrow BH) \equiv \sum_{i,j} \int_0^1 2zdz \int_0^{(x_{min} M_D)^2} du \int_0^1 \frac{dv}{v} f_i(v, Q) f_j(u/v, Q) \sigma_{i,j \rightarrow BH}(M_{BH} = us), \]

Figure 3: Total black hole cross section in femtobarns, including (solid curves) and not including (dashed) inelasticity as a function of \( M_D \) for ADD with \( n = 6 \) and \( M \) for RS1. The different curves from highest to lowest correspond to \( x_{min} = 1 - 6 \).
Upshot

- Black hole production threshold ($M_{BH}$) higher than originally thought
- **Means**
  - Lower production cross section
  - Lower reach in black hole mass
  - Lower entropy reach as well
- **Don’t produce classical thermal black holes**
  - What do we produce?
    - How many particles are in the final state?
- **$<N>$ calculation not necessarily reliable in quantum regime**
  - Nonetheless, use classical calculation as guide
  - Even if untrustworthy...Conclusion obvious
Figure 4: In the upper plots curves of total cross section for having 6 or more particles, including (solid curves) and not including (dashed) inelasticity as a function of $M_D$ for ADD with $n=6$ and $M$ for RS1. The different curves from highest to lowest correspond to $x_{\min} = 1 - 6$. In the lower plots the same curves are plotted for having 2 particles instead of 6 or more.

➢ Low multiplicity final states dominate and be worthy of study
Figure 5: Curves of constant 1 femtobarn cross section including the effects of inelasticity and a probability for getting either 2 particles (thicker curve) or greater than 6 particles (thin curve). In the left hand panel the curves are for ADD with 6 extra dimensions and are plotted as a function of $x \equiv M_{BH}/M_D$ and $M_D$. In the right hand panel the curves are for RS1 as a function of $x \equiv M_{BH}/M$ and $M$. 
What this Means

- Even 6 particle production cross section has markedly lower reach than 2 particle state
- Furthermore we don’t trust 6 particle states to be real multiparticle thermal state anyway!
- Not making classical black holes...
Face facts!

- Actually quite interesting
- Study 2 body final states: jets and leptons
- Can they be distinguished from background?
- Yes! For jets, transversality is key.
  - QCD dominated by t-channel exchange: forward
  - Black hole events isotropic—larger transverse xsection
- Plus clues from lepton vs jets
Suggested Search

- 2 jet, 2 lepton final states
- FROM WHATEVER WAS THERE!
- Measure:
  - Cross section
  - Angular dependence
    - Take advantage of t-channel SM cross section
- Look for distinctive characteristics
  - Energy and angular dependence
- We consider different quantum gravity models for what we might observe for these two functions
Compositeness Searches for Quantum Gravity

- Measure 2 functions of energy:
  - Differential cross section
  - Angular dependence through $R_\eta$ (much less systematic error)

\[
R_\eta \equiv \frac{N_{\text{events}}(0 < |\eta| < .5)}{N_{\text{events}}(.5 < |\eta| < 1)}
\]

- $R=0.6$ for SM
Clarification

- We don’t really think we can make precise predictions
- We use models for quantum gravity to understand possible predictions
- Take advantage of potentially rich data
- Ask: what are distinguishing features that experimentally probe quantum gravity
- Also note we forbid global quantum number violating transitions so we focus on B-conserving jets and lepton-number conserving processes
Figure 6: In the upper plots $d\sigma/dM_{jj}$ (units of pb/GeV) vs $M_{jj}(\text{TeV})$ is plotted for the case of SM QCD background, and a n=6 ADD model "black hole" behavior with $M_D=1,2,3,4$ TeV and $x_{min} = 1$ in the lefthand plot and a RS1 black hole behavior with $M = 1, 2, 3, 4$ TeV and $x_{min} = 1$ in the righthand plot. For other values of $x_{min}$ the curves simply start at the corresponding dijet mass. In the lower two plots the $R_\eta$ is plotted for the same parameters.
Distinctive Features

- Sudden turn on of cross section
- And transversality
- Even with slower turn on distinguishable
- Interference can distinguish among Ms
Alternative Model of QG: Weakly Coupled String Theory

- **Model**: Veneziano amplitude
- **Expect resonance behavior**
- **Then dramatic drop in transverse cross section**
  - Can readily distinguish from $Z'$
  - Can distinguish among different forms for Veneziano amplitude

\[
A^9_{ST} \equiv \frac{\Gamma \left(1 - \frac{s}{M_S^2}(1 + i\gamma)\right) \Gamma \left(1 - \frac{t}{M_S^2}(1 + i\gamma)\right)}{\Gamma \left(2 - \frac{s}{M_S^2}(1 + i\gamma) - \frac{t}{M_S^2}(1 + i\gamma)\right)}
\]

\[
A_{pp-\gamma j} \equiv A_{SM} A_{ST} = \frac{\Gamma \left(1 - \frac{s}{M_S^2}(1 + i\gamma)\right) \Gamma \left(1 - \frac{t}{M_S^2}(1 + i\gamma)\right)}{\Gamma \left(1 - \frac{s}{M_S^2}(1 + i\gamma) - \frac{t}{M_S^2}(1 + i\gamma)\right)}
\]
Stringy Results

Figure 8: In the upper plots $d\sigma/dM_{jj}$ (units of pb/GeV) vs $M_{jj}$ (TeV) is plotted for the case of SM QCD background (thicker curve), and a toy stringy behavior with $M_s=1$ TeV in the lefthand plot with $\gamma = .1, .3$ and $M_s=3$ TeV in the righthand plot with $\gamma = .1, .3, .6$. In the lower two plots the $R_\eta$ is plotted for the same parameters.
Can Distinguish Models

Vs. $Z'$

Figure 10: In the left plot $d\sigma/dM_{jj}$ (units of pb/GeV) vs $M_{jj}(TeV)$ is plotted for the case of SM QCD background, a toy stringy behavior with $M_s=3$ TeV and $\gamma = 0.2$ and a massive colored octet resonance (thicker curve) with mass and width chosen to mimic the differential cross section behavior near the resonance. In the right hand plot the same curves are plotted for $R_{qq}$, note the easily discernible difference between field theory resonance and “string” theory resonance.
Strings vs. Black Holes

- Black hole threshold is $M_s / g_s^2$
- String threshold is $M_s / g_s$
- String resonances only visible for small $g_s$
- Furthermore string bound might already be a few TeV (Antoniadis)
  - Model dependent so maybe more accessible range
- If you see strings, you certainly won’t see black holes (at LHC energies!)
Alternative QG: Higher Dimension Operators

- Black holes: we assumed high energy turn on and distinctive energy dependence
- Alternative: higher dimension operators
- Possible Sources:
  - Black Holes
  - Multigraviton exchange
  - High scale string theory (4 fermion for original Veneziano amplitude)
  - String theory in warped space
Distinctive Features for Higher Dimension Operators

- Energy dependence
- Threshold behavior: extend to low energies
- Spin structure
- Spacetime symmetries
- Charge and gauge structure

\[
\frac{c}{\Lambda^2} \sum (\bar{f} \gamma^\mu f)^2.
\]
Existing Bounds?

- LEP bounds on quark-lepton operators very strong: ~7 TeV (without $4\pi$)
- Can calculate loop gravity effects that generate such four-fermion operators
- Seems to preclude strong scale without a lower cutoff (such as string theory)- Guidice, Strumia
- However, brane width is also a natural cutoff since fermions necessarily on branes when black hole production conceivable
  - $\Lambda^{2n+2}$ vs. $\Lambda^2 \Lambda_{KK}^n$
- Allows lower quantum gravity scales
Result: 4-fermion

Cross section

\[ \Lambda = 1, 2, 4\, \text{TeV} \]

(c=1)
Distinguishing Characteristics if Related to Gravity

- Energy dependence
- Energy onset
- Leptons vs. Jets
  - Assuming neutral state 10% for thermal black holes vs. 20% for four-fermion operators
- More dramatic: do we produce neutral black hole—does it shed charge?
  - If yes, lepton rate much higher
  - SM lepton final state from q qbar initial pdf
Lepton cross section might be key

- Four-fermion operators: large lepton suppression
- Pdf, $\alpha$, u/s

Figure 13: $d\sigma/dM$ (units of pb/GeV) vs two body invariant mass (TeV) is plotted for QCD (the lowest curve) and a set of four fermion operators with $\Lambda = 1, 2, 4$ TeV for dijets in the upper curves. In the lower curves SM Drell-Yan production of leptons is plotted in combination with a four fermion operator that generates a $l^+ l^-$ final state with various $\Lambda = 1, 2, 4$ TeV.
From Black Holes

Much higher cross section since large fraction with larger pdfs

Even just losing u/s, alpha

Figure 15: $d\sigma/dM$ (units of pb/GeV) vs two body invariant mass(TeV). The curves from top to bottom represent black hole cross section for $M_D = 1$ TeV, $n = 6$, and $x_{mn} = 1$ for a black hole decaying into $l^+l^-$ (assuming any initial gauge charge is radiated softly), black hole cross section for $M_D = 1$ TeV and $x_{\min} = 1$ for a “charged” black hole decaying into $l\nu$, and the lowest curve is the Drell Yan background.
Summary

- Black holes not as “spectacular” as advertised

  BUT

- Lots of information about quantum gravity buried in 2→2!
- Initial increase in rate for more central processes always occurs
- R behavior: bh, string resonances, different forms for string, Z’ all distinctive
- Threshold behavior where interference matters
For the Future

- **Experiment**
  - Energy-dependent angle studies in dijets
  - Don’t assume particular form
- **Phenomenology**
  - Monte Carlos: allow decay to solely to 2 particles
  - Maybe remnant all there is!
  - Shedding of charge in initial state?
  - Different possible distributions of final states
- **Theory**
  - String theory models
- **Remain:**
  - Charge, spin
  - Signatures with missing energy, multibody final states (lepton and jet or more jets),
    - Compositeness type studies might be key
- **Get as much info at as high energies as possible**
- **Compositeness studies rich!**